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ZERO-ONE DOUBLE ACCEPTANCE SAMPLING PLAN FOR TRUNCATED LIFE TESTS BASED ON INVERSE RAYLEIGH DISTRIBUTION

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ABSTRACT

This paper proposes a zero one double sampling plan for the truncated life tests assuming that the lifetime of the product follows an inverse Rayleigh distribution. The minimum sample sizes necessary to ensure the specified life percentile are obtained for the specified consumer's confidence level. The operating characteristic values of the designed sampling plan and the minimum percentile ratios for the specified producer's risk are obtained. Numerical illustration is provided to explain the use of constructed tables.

KEYWORDS: Zero one Double Sampling Plan, Truncated Life Tests, Producers Risk, Consumer's Confidence Level and Operating Characteristic Function

INTRODUCTION

Products or items have variations even though they are produced by the same producer, same machine and under the same manufacturing conditions. The producer and the consumer are subject to risks due to the decision on the acceptance or rejection of lot of products based on sample results. Consumer's risk of accepting bad lots and Producer's risk of rejecting good lots may be minimized to a certain level by increasing the sample size. But this will increase the cost of inspection. Therefore an efficient acceptance sampling with truncation of test time is considered.

Several authors considered the design of acceptance sampling plan based on the population mean for truncated life tests. Epstein (1954) developed an acceptance sampling procedure for truncated life tests in the exponential case. Goode and Kao (1961) developed the sampling plan using Weibull distribution. Gupta and Groll (1961) developed the truncated life test sampling plan with the Gamma distribution. Kantam and Rosaiah (1998) developed the acceptance sampling plan based on half logistic distribution. Tsai and Wu (2006) developed the acceptance sampling plan for truncated life tests for generalised Rayleigh distribution. Lio et al., (2010) considered acceptance sampling plan for truncated life tests under the Birnbaum-Saunders distribution based on percentile.

Acceptance sampling plan based on mean may not satisfy the engineering requirements. This initiates the development of sampling plan using percentile life times than. Rao and Kantam (2010) developed acceptance sampling plans for truncated life tests based on the log-logistic distribution using percentile, Rao et.al (2012) using the inverse Rayleigh distribution. Muthulakshmi and Kavitha (2013) proposed single sampling plan for life test when the product life time has generalised Log-logistic distribution

The purpose of this paper is to design a zero one double sampling plan for industrial practitioners in testing of electronic components when they have strong evidence that the failure time follows inverse Rayleigh distribution, which has a wide spread application in survival analysis and reliability theory. The minimum sample sizes of the first and the second samples for proposed life test plan are determined using the specified consumer's confidence level by incorporating minimum average sample number. The operating characteristic values are analysed and the minimum percentile ratios of life time are obtained for the specified producers risk. Numerical illustrations are provided to explain its applicability.

INVERSE RAYLEIGH DISTRIBUTION

Assume that the lifetime of a product follows inverse Rayleigh distribution. The probability density function and the cumulative distribution function of inverse Rayleigh distribution are given by

$$f(t;\sigma) = \frac{2\sigma^2}{t^3}\, e^{-(\sigma/t)^2}; t \geq 0, \sigma > 0 \tag{1}$$

$$\mathbf{F}(t;\sigma) = e^{-(\sigma/t)^2}; \quad t \geq 0, \sigma > 0 \tag{2}$$

where σ is the scale parameter.

For 0<q<1, the 100qth percentile is given by $t_q = \sigma\,(-\ln q)^{-1/2}$

Let $\eta = (-\ln q)^{-1/2}$ then $\sigma = \frac{t_q}{\eta}$. This implies increase in q increases t_q

The cdf of the inverse Rayleigh distribution becomes

$$\mathrm{F}(t) = e^{-[(\frac{tq}{\eta}/t)^2]}; \quad t > 0 \tag{3}$$

it can be expressed as

$$F(t) = e^{-\left(\frac{1}{\eta\delta}\right)^2}, t > 0 \text{ when } \delta = t/t_q$$

Taking partial derivatives with respect to δ, we have

$$\frac{\partial F(t;\delta)}{\partial\delta} = \frac{2}{\eta\delta^3} e^{-(\frac{1}{\eta\delta})^2} \; ; t > 0$$

The acceptance sampling plan for percentiles under a truncated life test is to set up the minimum sample sizes for the given percentile such that the consumer's risk, the probability of accepting a bad lot, does not exceed 1- P*. Thus, the chance of rejecting a bad lot with $t_q < t_q^0$ is at least equal to P*. Therefore, for a given P*, the proposed acceptance sampling plan can be characterized by $(n_1, n_2, t/t_q^0)$.

DESIGN OF THE ZERO-ONE DOUBLE SAMPLING PLAN

The operating procedure of zero-one double sampling plan for the truncated life test has the following steps:

Step 1: Select a random sample of size n_1 from the submitted lot and put on test for preassigned experimental time t_0. Let d_1 be the number of failures. If $d_1=0$, accept the lot. If $d_1 \geq 2$, reject the lot.

Step 2: If $d_1=1$ then select a second random sample of size n_2 and let d_2 be the number of failures. If $d_2=0$, accept the lot otherwise reject the lot.

Under the proposed zero-one double acceptance sampling plan the probability of lot acceptance under binomial model is

$$P_a = (1-p)^{n_1}\left[1+n_1 p(1-p)^{n_2-1}\right] \tag{4}$$

where p is the probability that an item fails before t_0, which is given by

$$p = e^{-\left(\frac{1}{\eta\delta}\right)^2} \text{ where } \delta = t/t_q$$

The minimum sample sizes n_1 and n_2 ensuring $t_q > t_q^0$ at the consumer's confidence level, P* may be obtained by us in

$$P_a = (1-p_0)^{n_1}\left[1+n_1 p_0(1-p_0)^{n_2-1}\right] \leq 1-P^* \tag{5}$$

where p_0 is the probability evaluated at δ_0 which is given by

$$p_0 = e^{-\left(\frac{1}{\eta\delta_0}\right)^2} \text{ where } \delta_0 = t/t_q^{\ 0}$$

Multiple value for the sample sizes n_1 and n_2 exist from (5). In order to get the optimal sample sizes the concept of minimum ASN is incorporated along with the equation (5). The determination of minimum sample sizes reduces to

Minimize $ASN = n_1 + n_2 p_0 (1-p)^{n_1-1}$

subject to $(1-p_0)^{n_1}\left[1+n_1 p_0(1-p_0)^{n_2-1}\right] \leq 1-P^*$

where n_1 and n_2 are integers with $n_2 \leq n_1$

Table 1 is constructed to present the minimum sample sizes for the first and second sample with specified P^* (=0.75,0.90,0.95,0.99),δ(=0.9,1.0,1.5,2.0,2.5,3.0,3.5) and percentile q (=0.05,0.1,0.15) under binomial and Poisson distributions. Table values reveal that

- Increase in P^* increases the sample sizes for a fixed q and δ
- Increase in δ decreases the sample sizes for a fixed q and P^*
- Increase in q decreases the sample sizes for a fixed P^* and δ.

Figure 1 shows that the first sample size increases as the value of q decreases, when the value of δ is small.

OPERATING CHARACTERISTIC VALUES

The Operating Characteristic function of the zero-one double sampling plan is given by

$$P_a = (1-p)^{n_1}\left[1 + n_1 p(1-p)^{n_2 - 1}\right] \qquad (6)$$

where $p = F(t;\delta)$. It should be noticed that $F(t;\delta)$ can be represented as a function of $\delta = \frac{t}{t_q}$. Therefore

$p = F(\frac{t}{t_q^0}\frac{1}{d_q})$ where $d_q = \frac{t_q}{t_q^0}$.

Table 2 gives the OC values derived from (6) for the zero one double sampling plan with q= 0.1for the given P^*under inverse Rayleigh distribution. It is seen that at higher values of t_q/t_q^0 the OC values increases to one more rapidly.

For inverse Rayleigh distribution it is seen that

- Increase in t_q/t_q^0 increases the OC values for fixed q
- Increase in $t/t_q^0$and P*decreases the OC values for fixed q

MINIMUM PERCENTILE RATIO

The producer's risk is the probability of rejecting the lot when $t_q > t_q^0$. At a specified confidence level P^*, the smallest values of d_q are obtained for various percentile values using equation (6) with minimum sample values established in Table1 for the producer risk less than or equal to 0.05 and presented in Table 3. The numerical values in Table 4 indicates that

- Increase in P* decreases the minimum percentile ratio for fixed q
- Increase in q increases the minimum percentile ratio
- Increase in t/t_q^0 decreases the minimum percentile ratio

USES OF TABLES

In this section, an example with real data set is given to illustrate zero one double sampling with truncated life test for a specified confidence level when the life time of a product follows inverse Rayleigh distribution.

As an illustration, consider the analyst / the producer who wants to know whether the life time of mobile chargers produced in abundance by a leading electronic industry is longer than or equal to 1000 hours at the confidence level of 0.99. In the course of testing he wants to stop the experiment at 900 hours. This leads to the experimental termination time of δ =0.9 and q = 0.1. The required analysis may be done by applying the proposed life test plan. For the above stated condition table 1 gives the minimum sample sizes as n_1=80 and n_2 = 76. According to the proposed plan the analyst has to select 80 items first and put them on test for 900 hours. If no failure occurs accept the lot and reject the lot and reject if more than one failure occurs during the experimental time. Otherwise a second sample of size 76 is to be drawn and put on test for 900 hours. The lot will be accepted if there are no failures from the second sample and is rejected otherwise.

CONCLUSIONS

In this paper a zero-one double sampling plan is developed for the truncated life tests when the life time of items follows an inverse Rayleigh distribution. Tables are constructed for selecting a sampling plan for a given situation. The proposed plan is quite flexible as this approach may be extended to the products whose life time distributions follow Gamma, exponential, Pareto and many more.

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APPENDIES

Table 1: Minimum Sample Sizes for Zero-One Double Sampling Plan Using Binomial Model

q	P*	δ						
		0.5	0.7	0.9	1.1	1.5	1.9	2.5
0.05	0.75	67,65	33,31	6,5	3,3	2,2	2,1	2,1
	0.9	100,97	49,47	9,6	4,4	3,2	3,1	2,2
	0.95	125,123	61,61	11,8	5,5	4,2	3,2	3,1
	0.99	186,174	91,84	16,10	8,4	5,4	4,3	4,1
0.1	0.75	28,27	16,16	4,4	3,1	2,1	2,1	2,1
	0.9	42,40	24,23	6,5	4,2	3,1	2,2	2,1
	0.95	53,47	30,28	8,4	4,4	3,2	3,1	2,2
	0.99	78,69	45,34	11,7	6,4	4,4	4,2	3,2
0.15	0.75	17,16	11,9	4,2	2,2	2,1	2,1	2,1

	0.9	25,24	16,13	5,4	3,2	2,2	2,1	2,1
	0.95	31,31	20,16	6,5	4,2	3,2	2,2	2,2
	0.99	47,35	29,24	9,5	5,4	4,2	3,3	3,2
Minimum Sample Sizes for Zero-One Double Sampling Plan Using Poison Model								
q	**P***	**δ**						
		0.5	**0.7**	**0.9**	**1.1**	**1.5**	**1.9**	**2.5**
0.05	0.75	68,64	34,31	7,5	4,3	3,2	3,1	3,1
	0.9	101,99	50,49	10,8	6,3	5,2	4,2	4,2
	0.95	127,121	63,59	12,11	7,5	6,3	5,3	4,4
	0.99	188,182	93,93	18,13	10,8	8,4	7,4	6,5
0.1	0.75	29,27	17,16	5,4	3,3	3,2	3,1	3,1
	0.9	43,42	25,25	7,7	5,3	4,3	4,2	4,1
	0.95	54,51	32,27	9,7	6,4	5,3	5,2	4,3
	0.99	80,76	47,39	13,12	9,4	7,5	7,2	6,3
0.15	0.75	18,15	12,9	4,4	3,2	3,1	3,1	2,2
	0.9	26,26	17,15	6,5	5,2	4,2	4,1	3,3
	0.95	33,30	21,20	8,5	6,2	5,2	4,4	4,3
	0.99	49,40	31,31	11,9	8,4	7,3	6,4	6,3

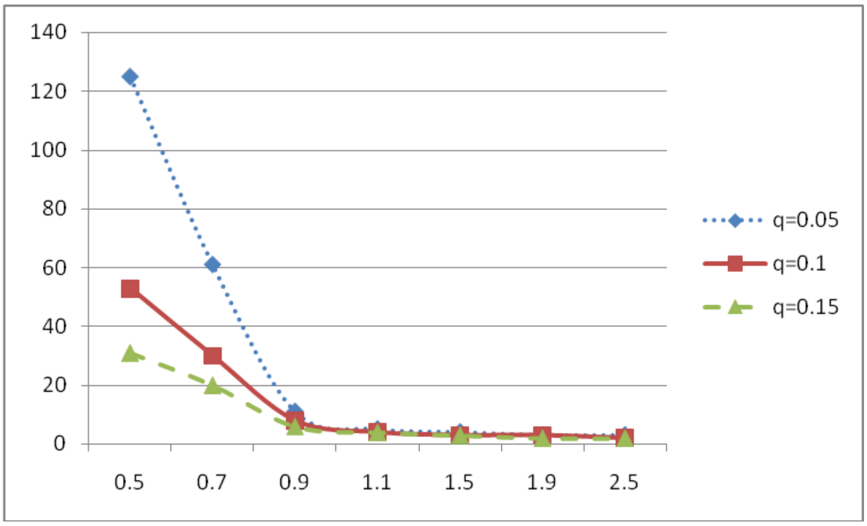

Figure 1: The First Sample Size vs Experiment Time at Confidence Level P*=0.95 for Inverse Rayleigh Distribution

Table 2: Operating Characteristic Values for Inverse Rayleigh Distribution q=0.1

P*	**t/t_q**	**n_1**	**n_2**	**t_q/t_q^0**					
				1	**1.5**	**1.75**	**2**	**2.5**	**2.75**
0.75	0.9	28	27	0.2499	0.997	0.9999	1	1	1
	1	16	16	0.2463	0.9893	0.9997	0.9999	1	1
	1.5	4	4	0.2321	0.8474	0.9644	0.9942	0.9999	0.9999
	2	3	1	0.2253	0.6975	0.8612	0.9477	0.9957	0.9991
	2.5	2	1	0.2264	0.5947	0.7536	0.8666	0.9721	0.9891
	3	2	1	0.1299	0.4069	0.5646	0.7054	0.8939	0.9434
	3.5	2	1	0.0781	0.2748	0.4069	0.5427	0.7727	0.8534
0.9	0.9	42	40	0.0093	0.9935	0.9999	1	1	1
	1	24	23	0.0986	0.9775	0.9994	0.9999	1	1
	1.5	6	5	0.9422	0.7406	0.9331	0.9886	0.9999	0.9999
	2	4	2	0.0728	0.4992	0.7388	0.8923	0.9902	0.9978
	2.5	3	1	0.0901	0.4132	0.6099	0.7731	0.9477	0.9791
	3	2	2	0.0688	0.2858	0.4415	0.5993	0.8419	0.9129
	3.5	2	1	0.0781	0.2748	0.4069	0.5427	0.7727	0.8534

0.95	0.9	53	47	0.0496	0.9903	0.9999	0.9999	1	1
	1	30	28	0.0498	0.9667	0.9991	0.9999	1	1
	1.5	8	4	0.0498	0.6815	0.9138	0.9849	0.9998	0.9999
	2	4	4	0.0436	0.3947	0.6549	0.8474	0.9851	0.9965
	2.5	3	2	0.0479	0.3109	0.5127	0.7009	0.9258	0.9696
	3	3	1	0.0382	0.2252	0.3799	0.5464	0.8159	0.8976
	3.5	2	2	0.0377	0.1727	0.2858	0.4185	0.6816	0.7863
0.99	0.9	78	69	0.0099	0.9799	0.9998	0.9999	1	1
	1	45	34	0.0099	0.9389	0.9982	0.9999	1	1
	1.5	11	7	0.0095	0.4973	0.8375	0.9689	0.9996	0.9999
	2	6	4	0.0091	0.2389	0.5125	0.7639	0.9745	0.9941
	2.5	4	4	0.0097	0.1323	0.2932	0.5015	0.8474	0.9332
	3	4	2	0.0044	0.0728	0.1735	0.3235	0.6668	0.8006
	3.5	3	2	0.0072	0.0688	0.1457	0.2581	0.5415	0.6765

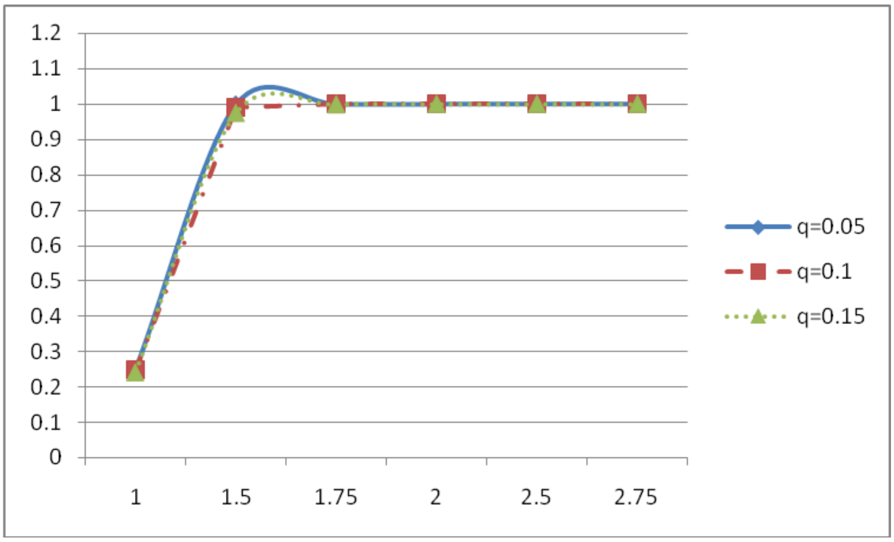

Figure 2: OC Curves for Inverse Rayleigh Distribution Under P*=0.75 and $t/t_q^0 = 1$

Table 3: Minimum Percentile Ratios of Zero-One Double Sampling Plan

q	P*	t/t_q						
		0.9	1.1	1.5	1.9	2.5	3	3.5
0.05	0.75	0.5843	0.5593	0.4597	0.3859	0.3364	0.2987	0.2547
	0.9	0.5643	0.5393	0.4397	0.3659	0.3164	0.2767	0.2387
	0.95	0.5543	0.5293	0.4297	0.3559	0.3066	0.2657	0.2347
	0.99	0.5393	0.5099	0.4099	0.3376	0.2878	0.2491	0.2251
0.1	0.75	0.7543	0.6393	0.5197	0.4387	0.3824	0.3187	0.2747
	0.9	0.6497	0.6193	0.4987	0.4087	0.3524	0.3037	0.2717
	0.95	0.6393	0.6087	0.4832	0.3939	0.3407	0.2947	0.2601
	0.99	0.6203	0.5832	0.4609	0.3707	0.3147	0.2691	0.2451
0.15	0.75	0.7599	0.7293	0.5769	0.4759	0.4042	0.3401	0.2899
	0.9	0.7293	0.6993	0.5437	0.4559	0.3864	0.3367	0.2887
	0.95	0.7143	0.6793	0.5297	0.4359	0.3666	0.3249	0.2721
	0.99	0.6943	0.6499	0.5039	0.4059	0.3466	0.2949	0.2621